Bits and Boolean Algebra

CC 110

Aristotelian Logic

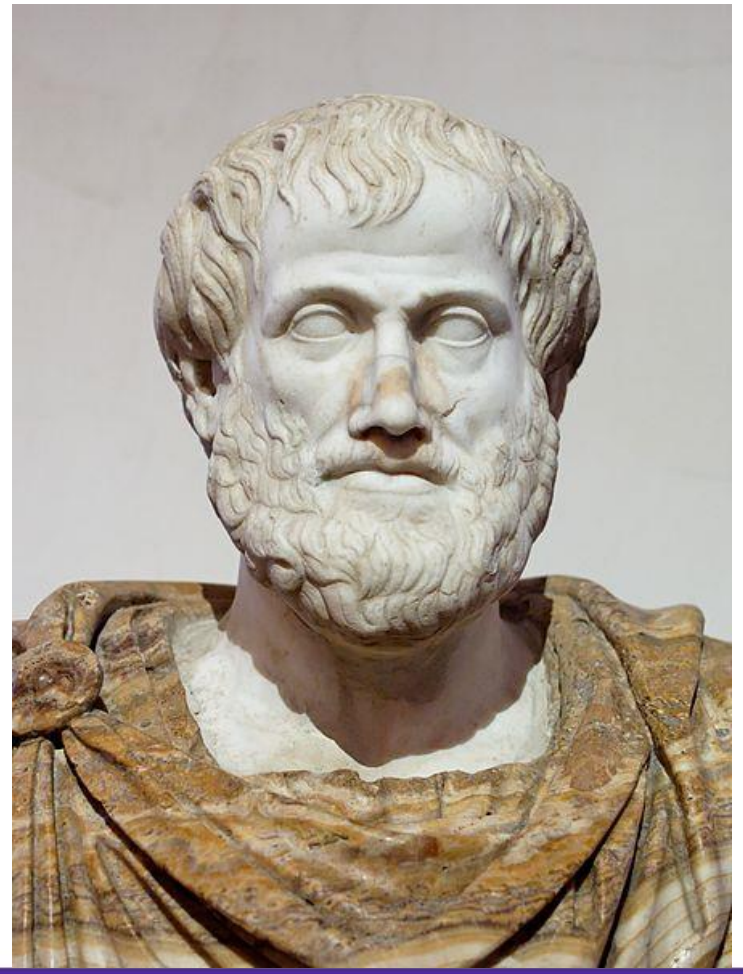
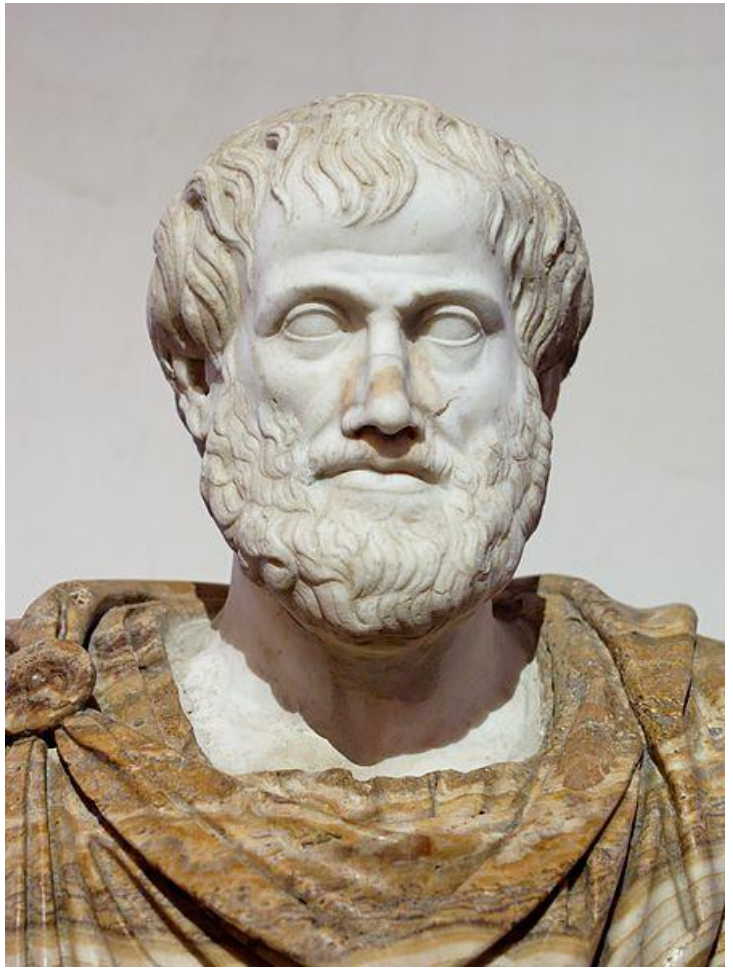


Image Source

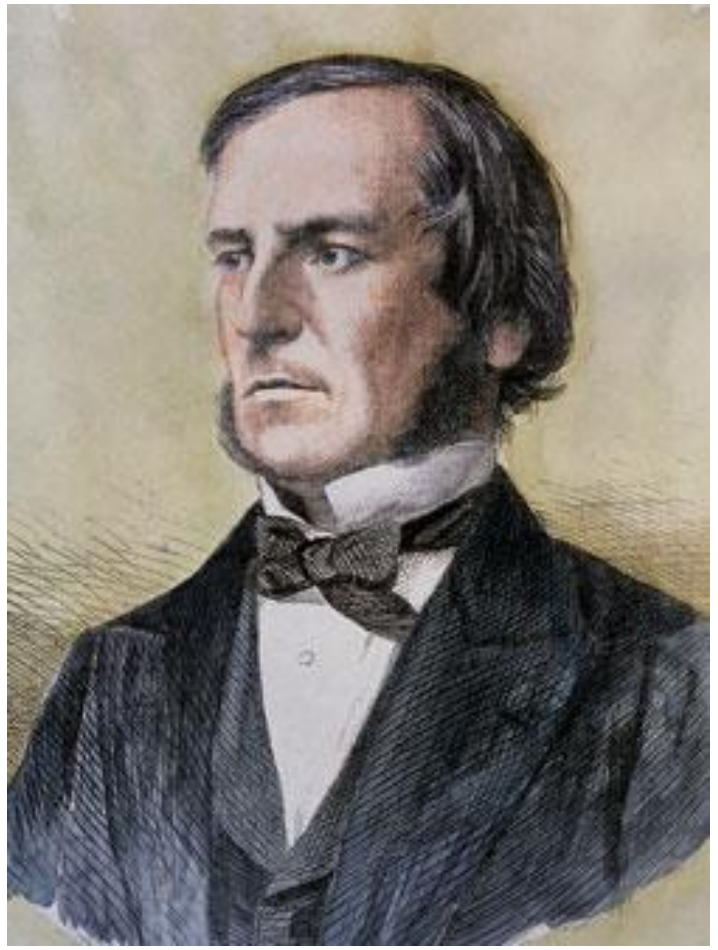
Aristotelian Logic

- Premise:
  - All humans are mortal
  - Socrates is a human
- Conclusion:
  - Therefore, Socrates is mortal



Boolean Logic

- The Laws of Thought
- Premise:
  - $A \wedge B$
  - B $\wedge C$
- Conclusion:
  - A $\wedge C$



Boolean Values

- Boolean
  - TRUE, FALSE
- Binary
  - 1, 0
- Electrical
  - ON, OFF
- These are traditional representations, but they can be reversed for various reasons, check the manual!

Boolean Operators

and

&& ∧

or

|| ∨

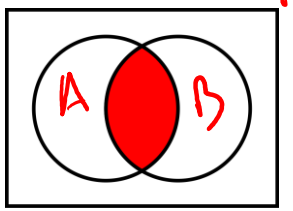
Exclusive Or (XOR)

⊕

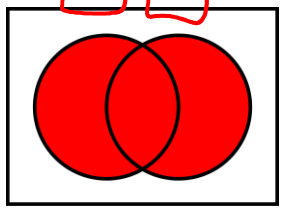
not

! ¬

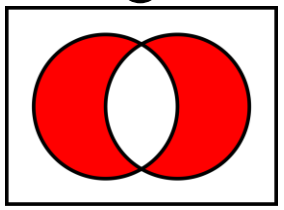
A ∧ B = True



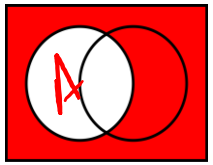
A ∨ B = True



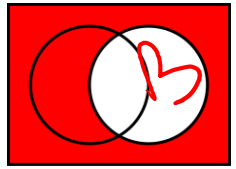
A ⊕ B

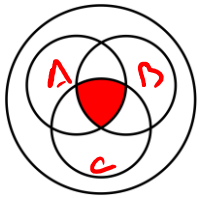


¬ A

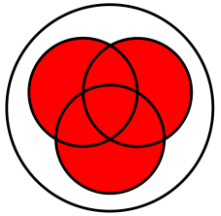


¬ B

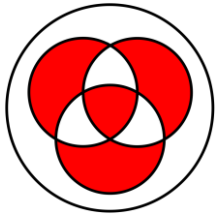




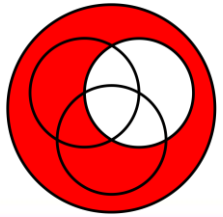
A ∧ B ∧ C



A ∨ B ∨ C



A ⊕ B ⊕ C



¬ B

De Morgan's Theorem

- Distribute the negative (¬) then swap ands (∧) and or's (∨)
- Negation (inverse) of a logic statement

  ¬ (A ∧ B) = (¬ A) ∨ (¬ B)

  ¬ (A ∨ B) = (¬ A) ∧ (¬ B)

Boolean Algebra

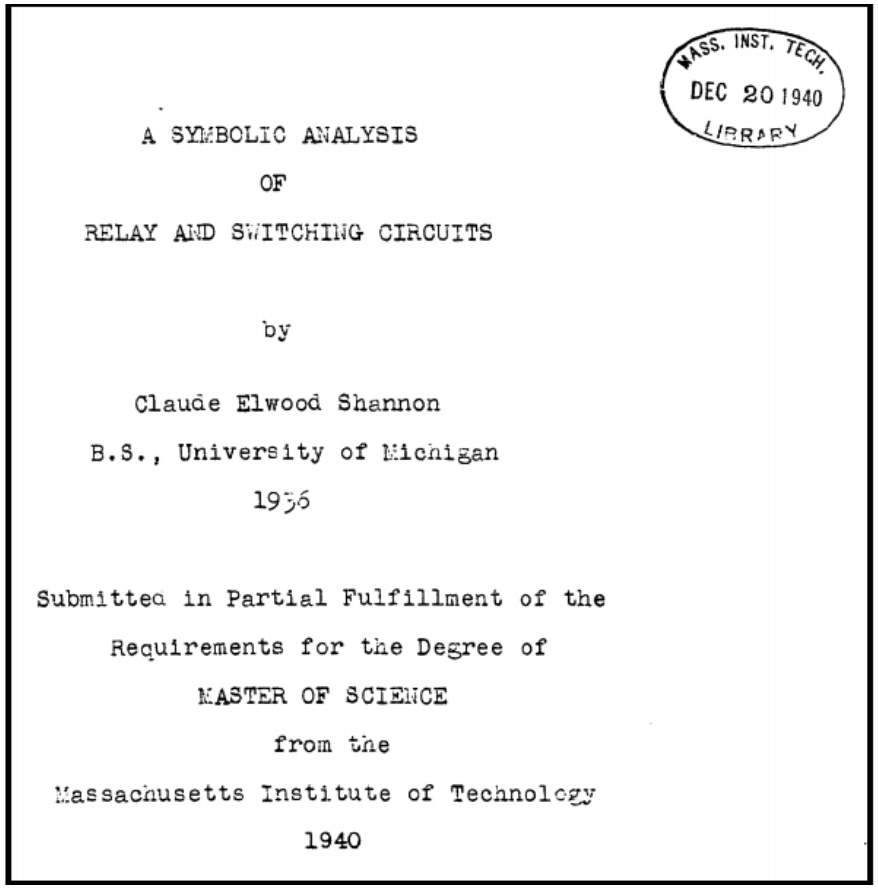
- $\vee$ works like addition (+)
- $\neg$ works like negation (−)
- $\wedge$ works like multiplication (×)
- Associative: $(A \wedge B) \wedge C = A \wedge (B \wedge C)$
- Commutative: $(A \wedge B) = (B \wedge A)$
- Distributive: $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$

Logic via Electrical Switches?

Charles Sanders Peirce

Claude Shannon

- **A Symbolic Analysis of Relay and Switching Circuits**



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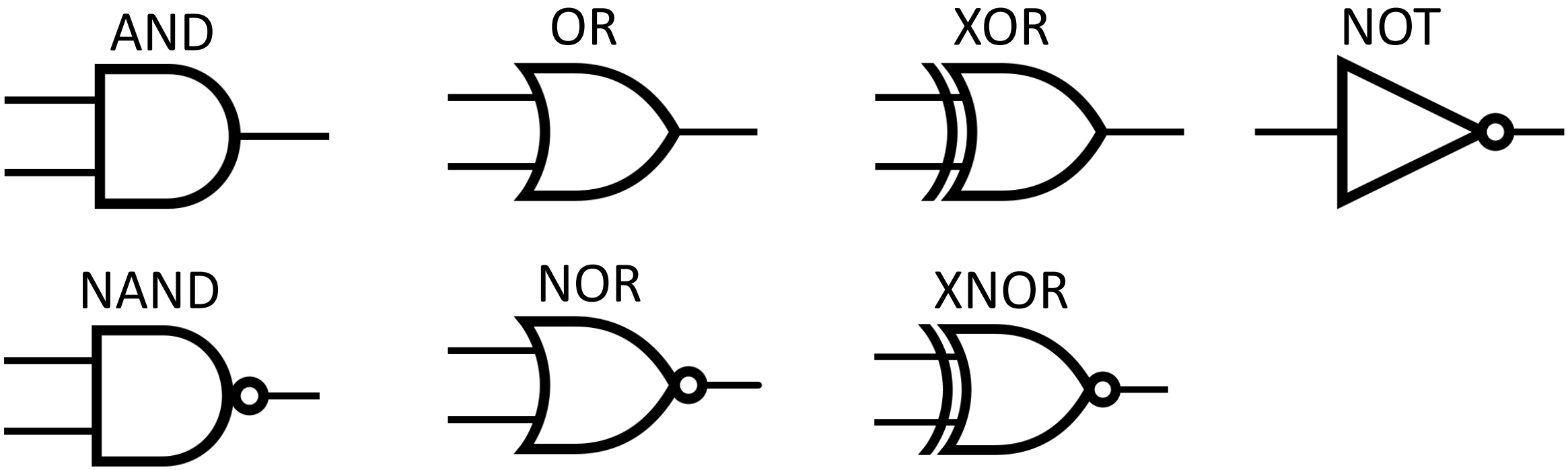
A SYMBOLIC ANALYSIS
OF
RELAY AND SWITCHING CIRCUITS

by

Claude Elwood Shannon
B.S., University of Michigan
1936

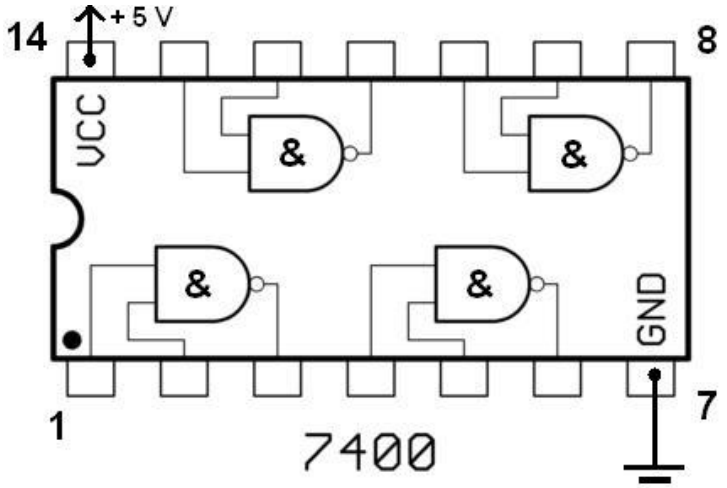
Submitted in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
from the
Massachusetts Institute of Technology
1940

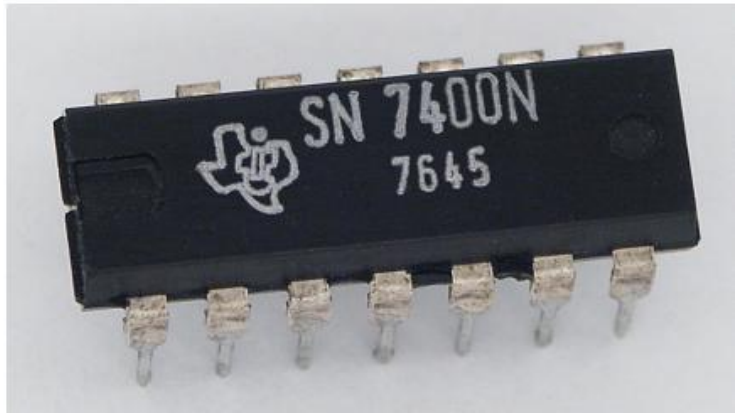
Logic Gates



Note: The little circle at the end of the NOT gate is the only part that matters.

Universal Logic Gates

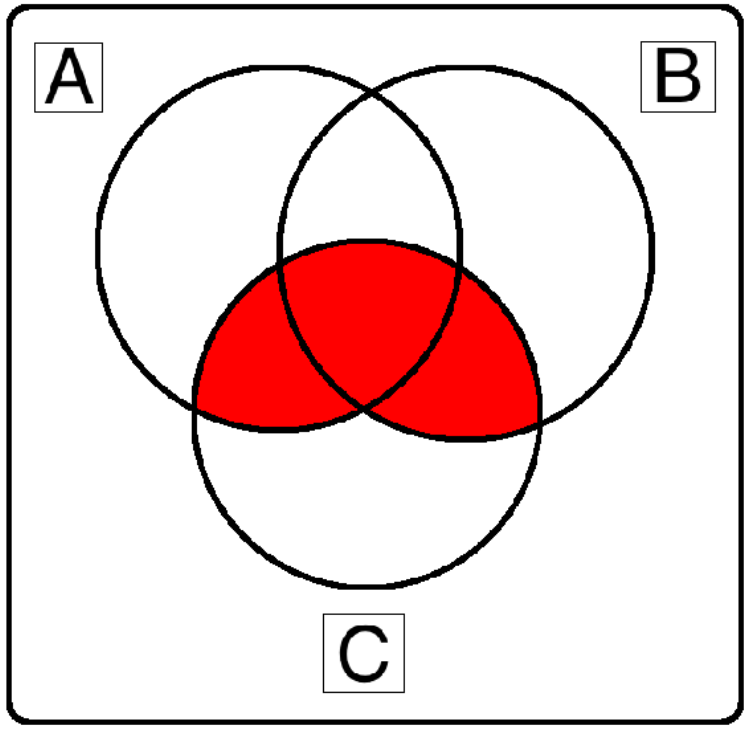


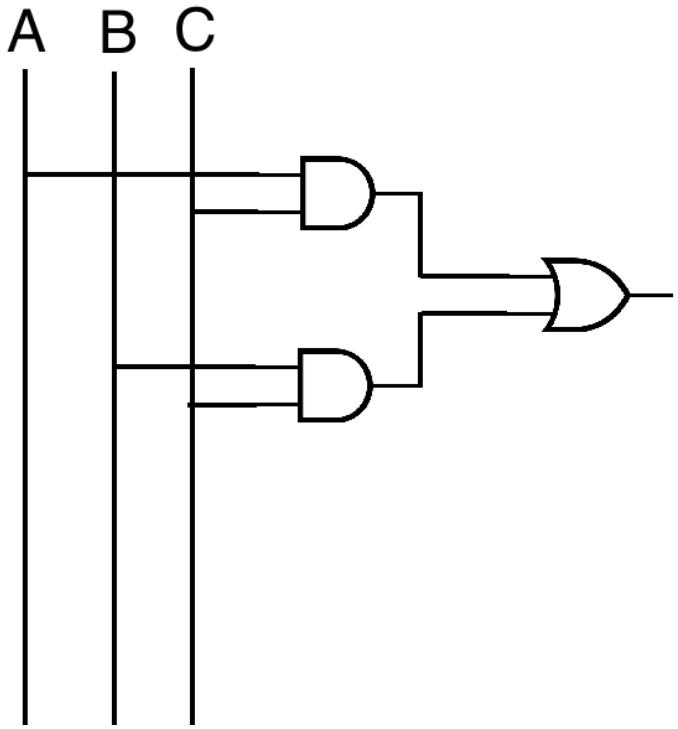


Example 1

A	B	C	OUT
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$(A \wedge C) \vee (B \wedge C)$





$C \wedge (A \vee B)$ works as well